Supplemental Material for "Light Structure from Pin Motion: Simple and Accurate Point Light Calibration for Physics-based Modeling"

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In this supplemental material

A. we derive the shadow formation equations (2) and (4) from the paper,
B. we derive the linear relaxation equations (7) and (9) from the paper,
C. we show estimation accuracies from a simulation experiment where we simultaneously applied shadow position noise *and* board pose noise and varied the number of poses N_p and number of casters N_c, and
D. we provide the calibration pattern for estimating the board pose.

In addition, we also submit a video that demonstrates how simple it is to build and use the proposed calibration target and procedure.

A Derivation of shadow formation equations (2) and (4)

A.1 Near point light – Equation (2)

Inserting $\mathbf{c} = [c_x, c_y, c_z]^\top$, $\bar{\mathbf{s}} = [s_x, s_y, 0]^\top$, and $\mathbf{l} = [l_x, l_y, l_z]^\top$ (all in non-homogeneous 3D global world coordinates) into Eq. (1) yields

$$(\mathbf{c} - \bar{\mathbf{s}}) \times (\mathbf{l} - \bar{\mathbf{s}}) = \begin{bmatrix} c_x - s_x \\ c_y - s_y \\ c_z - 0 \end{bmatrix} \times \begin{bmatrix} l_x - s_x \\ l_y - s_y \\ l_z - 0 \end{bmatrix} = \mathbf{0}.$$

Expanding the cross-product yields

$$\begin{cases} (c_y - s_y)l_z - c_z(l_y - s_y) = 0, \\ c_z(l_x - s_x) - (c_x - s_x)l_z = 0, \\ (c_x - s_x)(l_y - s_y) - (c_y - s_y)(l_x - s_x) = 0, \end{cases}$$

From this it follows that

$$\begin{cases} s_x = \dfrac{c_x l_z - c_z l_x}{l_z - c_z}, \\ s_y = \dfrac{c_y l_z - c_z l_y}{l_z - c_z}. \end{cases}$$

We can then write $\mathbf{s}$ in homogeneous coordinates using a scaling parameter γ:

$$\gamma\hat{\mathbf{s}} = \begin{bmatrix} \frac{c_x l_z - c_z l_x}{l_z - c_z} \\ \frac{c_y l_z - c_z l_y}{l_z - c_z} \\ 1 \end{bmatrix}$$

$$\iff \underbrace{\gamma(l_z - c_z)}_{-\lambda}\,\hat{\mathbf{s}} = \begin{bmatrix} c_x l_z - c_z l_x \\ c_y l_z - c_z l_y \\ l_z - c_z \end{bmatrix} = \underbrace{\begin{bmatrix} l_z & 0 & -l_x & 0 \\ 0 & l_z & -l_y & 0 \\ 0 & 0 & -1 & l_z \end{bmatrix}}_{-\mathbf{L}} \underbrace{\begin{bmatrix} c_x \\ c_y \\ c_z \\ 1 \end{bmatrix}}_{\hat{\mathbf{c}}}. \quad \square$$

Further, the decomposition of $\mathbf{L}$ into an intrinsic and an extrinsic matrix can be verified by multiplication of the intrinsic and the extrinsic matrices.

A.2 Distant light – Equation (4)

Inserting $\mathbf{c}$, $\bar{\mathbf{s}}$, and $\mathbf{l}$ into Eq. (3) yields

$$(\mathbf{c} - \bar{\mathbf{s}}) \times \mathbf{l} = \begin{bmatrix} c_x - s_x \\ c_y - s_y \\ c_z - 0 \end{bmatrix} \times \begin{bmatrix} l_x \\ l_y \\ l_z \end{bmatrix} = \mathbf{0}.$$

Expanding yields

$$\begin{cases} (c_y - s_y)l_z - c_z l_y = 0, \\ c_z l_x - (c_x - s_x)l_z = 0, \\ (c_x - s_x)l_y - (c_y - s_y)l_x = 0, \end{cases}$$

from which it follows that

$$\begin{cases} s_x = \dfrac{c_x l_z - c_z l_x}{l_z}, \\ s_y = \dfrac{c_y l_z - c_z l_y}{l_z}. \end{cases}$$

We can then write $\mathbf{s}$ in homogeneous coordinates using a scaling parameter γ:

$$\gamma\hat{\mathbf{s}} = \begin{bmatrix} \frac{c_x l_z - c_z l_x}{l_z} \\ \frac{c_y l_z - c_z l_y}{l_z} \\ 1 \end{bmatrix}$$

$$\iff \underbrace{\gamma l_z}_{-\lambda}\,\hat{\mathbf{s}} = \begin{bmatrix} c_x l_z - c_z l_x \\ c_y l_z - c_z l_y \\ l_z \end{bmatrix} = \underbrace{\begin{bmatrix} l_z & 0 & -l_x & 0 \\ 0 & l_z & -l_y & 0 \\ 0 & 0 & 0 & l_z \end{bmatrix}}_{-\mathbf{L}} \underbrace{\begin{bmatrix} c_x \\ c_y \\ c_z \\ 1 \end{bmatrix}}_{\hat{\mathbf{c}}}. \quad \square$$

B Derivation of linear relaxation equations (7) and (9)

B.1 Near point light – Equation (7)

With $\mathbf{c}_j = [c_{j,x}, c_{j,y}, c_{j,z}]^\top$, $\bar{\mathbf{s}}_{ij} = [s_x, s_y, 0]^\top$, $\mathbf{R}_i^\top = \begin{bmatrix} r_0 & r_1 & r_2 \\ r_3 & r_4 & r_5 \\ r_6 & r_7 & r_8 \end{bmatrix}$, $\mathbf{l} = [l_x, l_y, l_z]^\top$, and $-\mathbf{R}_i^\top \mathbf{t}_i = [t_x, t_y, t_z]^\top$, we can rewrite Eq. (6) as

$$(\mathbf{c}_j - \bar{\mathbf{s}}_{ij}) \times (\mathbf{R}_i^\top \mathbf{l} - \mathbf{R}_i^\top \mathbf{t}_i - \bar{\mathbf{s}}_{ij}) = \begin{bmatrix} c_{j,x} - s_x \\ c_{j,y} - s_y \\ c_{j,z} \end{bmatrix} \times \begin{bmatrix} [r_0, r_1, r_2]\,\mathbf{l} + t_x - s_x \\ [r_3, r_4, r_5]\,\mathbf{l} + t_y - s_y \\ [r_6, r_7, r_8]\,\mathbf{l} + t_z \end{bmatrix} = \mathbf{0}.$$

Expanding the cross-product yields

$$\begin{cases} (c_{j,y} - s_y)([r_6, r_7, r_8]\,\mathbf{l} + t_z) - c_{j,z}([r_3, r_4, r_5]\,\mathbf{l} + t_y - s_y) = 0, \\ c_{j,z}([r_0, r_1, r_2]\,\mathbf{l} + t_x - s_x) - (c_{j,x} - s_x)([r_6, r_7, r_8]\,\mathbf{l} + t_z) = 0, \\ (c_{j,x} - s_x)([r_3, r_4, r_5]\,\mathbf{l} + t_y - s_y) - (c_{j,y} - s_y)([r_0, r_1, r_2]\,\mathbf{l} + t_x - s_x) = 0, \end{cases}$$

which we can rewrite as

$$\begin{cases} -s_y t_z = -c_{j,y}([r_6,r_7,r_8]\,\mathbf{l} + t_z) + s_y\,[r_6,r_7,r_8]\,\mathbf{l} + c_{j,z}([r_3,r_4,r_5]\,\mathbf{l} + t_y - s_y), \\ s_x t_z = -c_{j,z}([r_0,r_1,r_2]\,\mathbf{l} + t_x - s_x) + c_{j,x}([r_6,r_7,r_8]\,\mathbf{l} + t_z) - s_x\,[r_6,r_7,r_8]\,\mathbf{l}, \\ -s_x t_y + s_y t_x = -c_{j,x}([r_3,r_4,r_5]\,\mathbf{l} + t_y - s_y) + s_x[r_3,r_4,r_5]\,\mathbf{l} + c_{j,y}([r_0,r_1,r_2]\,\mathbf{l} + t_x - s_x) - s_y[r_0,r_1,r_2]\,\mathbf{l}, \end{cases}$$

which can then in turn be rewritten in the matrix form of Eq. (7).

B.2 Distant point light – Equation (9)

Keeping the definitions of $\mathbf{c}_j$, $\bar{\mathbf{s}}_{ij}$, $\mathbf{R}_i^\top$, and $-\mathbf{R}_i^\top \mathbf{t}_i$, we can rewrite Eq. (8) as

$$(\mathbf{c}_j - \bar{\mathbf{s}}_{ij}) \times \mathbf{R}_i^\top \mathbf{l} = \begin{bmatrix} c_{j,x} - s_x \\ c_{j,y} - s_y \\ c_{j,z} \end{bmatrix} \times \begin{bmatrix} [r_0, r_1, r_2]\,\mathbf{l} \\ [r_3, r_4, r_5]\,\mathbf{l} \\ [r_6, r_7, r_8]\,\mathbf{l} \end{bmatrix} = \mathbf{0}.$$

Expanding the cross-product yields

$$\begin{cases} (c_{j,y} - s_y)\,[r_6, r_7, r_8]\,\mathbf{l} - c_{j,z}\,[r_3, r_4, r_5]\,\mathbf{l} = 0, \\ c_{j,z}\,[r_0, r_1, r_2]\,\mathbf{l} - (c_{j,x} - s_x)\,[r_6, r_7, r_8]\,\mathbf{l} = 0, \\ (c_{j,x} - s_x)\,[r_3, r_4, r_5]\,\mathbf{l} - (c_{j,y} - s_y)\,[r_0, r_1, r_2]\,\mathbf{l} = 0. \end{cases}$$

After setting $\mathbf{l} = [l_x, l_y, 1]^\top$, we can rewrite this as

$$\begin{cases} (c_{j,y} - s_y)(r_6 l_x + r_7 l_y + r_8) - c_{j,z}(r_3 l_x + r_4 l_y + r_5) = 0, \\ c_{j,z}(r_0 l_x + r_1 l_y + r_2) - (c_{j,x} - s_x)(r_6 l_x + r_7 l_y + r_8) = 0, \\ (c_{j,x} - s_x)(r_3 l_x + r_4 l_y + r_5) - (c_{j,y} - s_y)(r_0 l_x + r_1 l_y + r_2) = 0, \end{cases} \quad \text{and}$$

$$\begin{cases} -s_y r_8 = -c_{j,y}(r_6 l_x + r_7 l_y + r_8) + s_y(r_6 l_x + r_7 l_y) + c_{j,z}(r_3 l_x + r_4 l_y + r_5), \\ s_x r_8 = -c_{j,z}(r_0 l_x + r_1 l_y + r_2) + c_{j,x}(r_6 l_x + r_7 l_y + r_8) - s_x(r_6 l_x + r_7 l_y), \\ s_y r_2 - s_x r_5 = -c_{j,x}(r_3 l_x + r_4 l_y + r_5) + s_x(r_3 l_x + r_4 l_y) + c_{j,y}(r_0 l_x + r_1 l_y + r_2) - s_y(r_0 l_x + r_1 l_y), \end{cases}$$

which can then be rewritten in the matrix form of Eq. (9).

C Simulation experiment for combined shadow position and board pose noise

In Sec. 4.1, we studied the effect of shadow position errors and board pose errors separately. In this section we show simulation results where we added both shadow position noise *and* board pose noise at the same time.

In the top rows of Figs. 7 and 8 we can see that shadow position noise causes errors roughly twice as big as those from board pose noise. In this experiment we thus set the standard deviation for shadow position noise to $\sigma_{\text{shadows}} = 0.01$ and for board pose noise to $\sigma_{\text{pose}} = 0.005$.

For the number of shadow casters varying from 1 to 9 and the number of poses varying from 5 to 100, Figure C.1 shows color-coded (log-scale) median error in the *top row* and standard deviation in the *bottom row*. Again, bundle adjustment and more poses and casters decrease the error. If the application at hand dictates one of the two parameters, *e.g.*, if time restrictions forbid increasing N_p beyond 20, this can always be countered by increasing the other parameter.

Even though the minimal conditions for solving the calibration are 1 caster and 4 or 5 poses, the data suggests that one should probably never use less than 3 casters and less than 10 or better yet 20 poses. For example the standard deviation for 1 or 2 casters and 5 poses is probably unbearably high. In the paper we showed that 5 casters and 20–50 poses (which can be obtained quickly in our experience) produce an accuracy superior to related work.

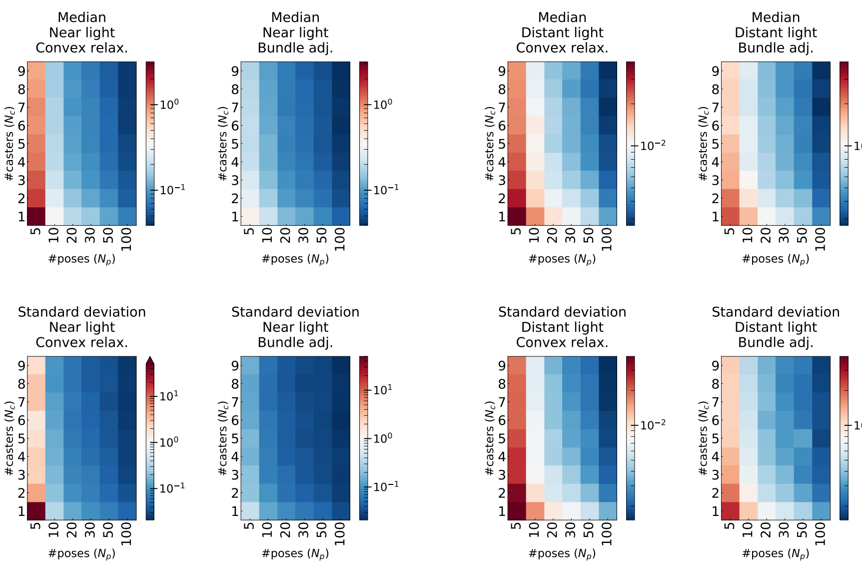

Fig. C.1. Estimation error for synthetic near and distant light with Gaussian noise added to shadow positions ($\sigma_{\text{shadows}} = 0.01$) and board orientation in degrees ($\sigma_{\text{pose}} = 0.005$). N_p is on the x-axis and N_c is on the y-axis. For each data point we performed 500 random trials. *Top row:* The median of the absolute error (near light)/angular error in degree (distant light). *Bottom row:* The error's standard deviation.